Algorithm AES – Rijndael

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1 Introduction

The **Rijndael Algorithm** is a block cipher, that encrypt blocks of 128, 192, or 256 bits using symmetric keys of 128, 192 or 256 bits. It consists of an initial round (`AddRoundKey`), and r standard rounds, r is 10,12 o 14 that depending on the block and key length. The first $r-1$ rounds are similar and they consist of 4 transformations, called:

- `ByteSub` – Substitution Bytes
- `ShiftRow` – Shift Rows
- `MixColumn` – Multiply columns
- `AddRoundKey` – Xor-ed by key

The last round only perform the transformations:

- `ByteSub`
- `ShiftRow`
- `AddRoundKey`

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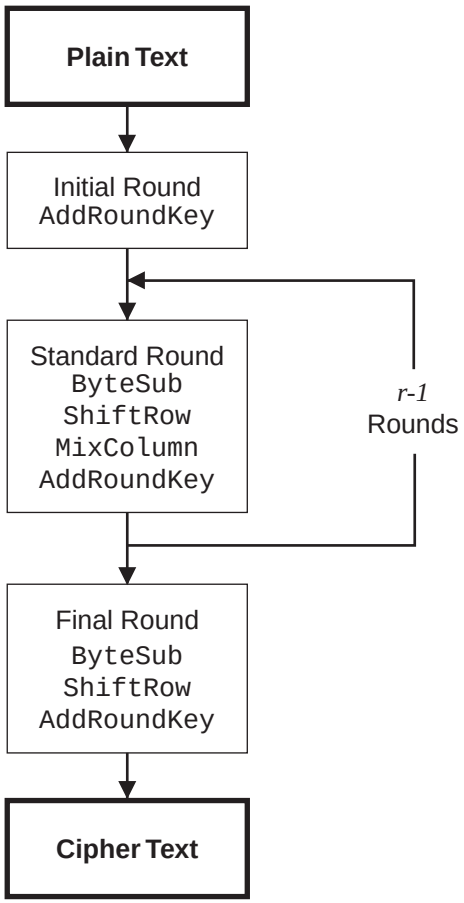
The whole process is shown in the following diagram:



The transformations are applied to an array B, in matrix form B_{ji}, that is mapping in the following form: (This is the 128 bits case or 16 bytes)

$$B_{00}B_{10}B_{20}B_{30}B_{01}B_{11}B_{21}B_{31}B_{02}B_{12}B_{22}B_{32}B_{03}B_{13}B_{23}B_{33}$$

and the matrix is:

$$\begin{pmatrix} B_{00} & B_{10} & B_{20} & B_{30} \\ B_{01} & B_{11} & B_{21} & B_{31} \\ B_{02} & B_{12} & B_{22} & B_{32} \\ B_{03} & B_{13} & B_{23} & B_{33} \end{pmatrix}$$

In the 192 bits case, we have to add 2 columns more and in the 256 bits case, we have to add 4 columns more.

Notice that in B_{ji}, j is the row and i the column.

All the transformations are applied to B, except `AddRoundKey` (B,K_r), that uses the sub-key $K_r[ji]$ as input.

One of the most important things in Rijndael is the key schedule: it generates the sub-keys K_r and load them into a linear array W. The length of the array W depends on the block and key length, and W provides the sub-keys K_r to every round r. Also the number of rounds r depends on the block and key length.

If at least the block or key length is 256 bits, then 14 rounds are needed. If both the block and key length are 128 bits, then 10 rounds are needed; otherwise 12 rounds are needed.

In this tutorial we will show step by step how Rijndael works, with a plain text B of 128 bits and a key K of 128 bits. The explanation is divided in two parts:

1. Key schedule to provide K_r.

2. Encryption (10 rounds in 128 bits case).

We are going to use the plain text $B = 000102030405060708090A0B0C0D0E0F$ and the key $K = 000102030405060708090A0B0C0D0E0F$, both in hexadecimal.

This is the first vector test in the file `ecb_iv.txt`, taken from `rijndael-vals.zip`, available at `http://csrc.nist.gov/encryption/aes/rijndael/`

In every part we are going to explain in detail how it works, so the outputs will be very ease to check, in a few steps.

2 Key schedule

The key schedule is used to provide the sub-keys K_r

Starting with the key K, we extend it and get the linear array W. W is an array with length $Nb \times (N_r + 1)$, where N_b is the number of columns of the matrix B_{ij} and N_r is the number of rounds.

In this case, $N_b = 4$ and $N_r = 10$, so W have $4 \times (10 + 1)] = 44$ words of 32b. Every block of 4 words have 128 bits and it is the sub-key K_r. The first sub-key is $K_0 = (W_0, W_1, W_2, W_3)$, that is the copy of the key K.

The next sub-keys are obtained from the K_0. The rule is the following

$$\begin{array}{rcl} W_4 & = & W_0 \text{ xor } temp_1 \\ W_5 & = & W_1 \text{ xor } W_4 \\ W_6 & = & W_2 \text{ xor } W_4 \\ W_7 & = & W_3 \text{ xor } W_4 \\ W_8 & = & W_4 \text{ xor } temp_2 \\ & \vdots & \\ W_{43} & = & W_{39} \text{ xor } W_{42} \\ W_{44} & = & W_{40} \text{ xor } temp_{11} \end{array}$$

Look at the cases of W_i where ($i \bmod N_k \neq 0$) and the rule is

$$W_i = W_{i-N_k} \text{ xor } W_{i-1}$$

with $N_r = 4$.

Where ($i \bmod N_k = 0$), we have

$$W_i = W_{i-N_k} \text{ xor } temp_k$$

where $temp_k = \texttt{ByteSub}(S_1 W_{i-1})$ xor $rcon_k$.

`ByteSub` is applied to W_{i-1} with a byte shifted, and $rcon_k$ is defined as $rcon_k = (RC_k, 00, 00, 00)$, with $RC_1 = 1$, $RC_k = X \times RC_{k-1} = X^{k-1}$ and $RC_k \in GF(2^8)$ for $k = i/4$ and $i = 4, 8, 12, \ldots, 44$.

Consider the initial sub-key

$$K_0 = 00\ 01\ 02\ 03\ 04\ 05\ 06\ 07\ 08\ 09\ 0A\ 0B\ 0C\ 0D\ 0E\ 0F$$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_4 = W_0 \text{ xor } \texttt{ByteSub}(S_1 W_3)$$

$$\begin{array}{rcl} D7 & = & 00 \text{ xor } D7 \\ AA & = & 01 \text{ xor } AB \\ 74 & = & 02 \text{ xor } 76 \\ FD & = & 03 \text{ xor } FE \end{array}$$

$$\begin{aligned} W_4[0] &= W_4[0] \text{ xor } rcon_1 \\ D6 &= D7 \text{ xor } 02 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{lll} W_5 = W_1 \text{ xor } W_4 & W_6 = W_2 \text{ xor } W_5 & W_7 = W_3 \text{ xor } W_6 \\ \\ D2 = 04 \text{ xor } D6 & DA = 08 \text{ xor } D2 & D6 = 0C \text{ xor } DA \\ AF = 05 \text{ xor } AA & A6 = 09 \text{ xor } AF & AB = 0D \text{ xor } A6 \\ 72 = 06 \text{ xor } 74 & 78 = 0A \text{ xor } 72 & 76 = 0E \text{ xor } 78 \\ FA = 07 \text{ xor } FD & F1 = 0B \text{ xor } FA & FE = 0F \text{ xor } F1 \end{array}$$

Sub-key $K_1 = D6\,AA\,74\,FD\,D2\,AF\,72\,FA\,DA\,A6\,78\,F1\,D6\,AB\,76\,FE$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_8 = W_4 \text{ xor } \texttt{ByteSub}(S_1 W_7)$$

$$\begin{aligned} B4 &= D6 \text{ xor } 62 \\ 92 &= AA \text{ xor } 38 \\ CF &= 74 \text{ xor } BB \\ 0B &= FD \text{ xor } F6 \end{aligned}$$

$$\begin{aligned} W_8[0] &= W_8[0] \text{ xor } rcon_2 \\ B6 &= B4 \text{ xor } 04 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{lll} W_9 = W_5 \text{ xor } W_8 & W_{10} = W_6 \text{ xor } W_9 & W_{11} = W_7 \text{ xor } W_{10} \\ \\ 64 = D2 \text{ xor } B6 & BE = DA \text{ xor } 64 & 68 = D6 \text{ xor } BE \\ 3D = AF \text{ xor } 92 & 9B = A6 \text{ xor } 3D & 30 = AB \text{ xor } 9B \\ BD = 72 \text{ xor } CF & C5 = 78 \text{ xor } BD & B3 = 76 \text{ xor } C5 \\ F1 = FA \text{ xor } 0B & 00 = F1 \text{ xor } F1 & FE = FE \text{ xor } 00 \end{array}$$

Sub-key $K_2 = B6\,92\,CF\,0B\,64\,3D\,BD\,F1\,BE\,9B\,C5\,00\,68\,30\,B3\,FE$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{12} = W_8 \text{ xor } \texttt{ByteSub}(S_1 W_{11})$$

$$\begin{aligned} B2 &= B6 \text{ xor } 04 \\ FF &= 92 \text{ xor } 6D \\ 74 &= CF \text{ xor } BB \\ 4E &= 0B \text{ xor } 45 \end{aligned}$$

$$\begin{aligned} W_{12}[0] &= W_{12}[0] \text{ xor } rcon_3 \\ B6 &= B2 \text{ xor } 08 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{aligned} W_{13} &= W_9 \text{ xor } W_{12} \\ D2 &= 64 \text{ xor } B6 \\ C2 &= 3D \text{ xor } FF \\ C9 &= BD \text{ xor } 74 \\ BF &= F1 \text{ xor } 4E \end{aligned} \qquad \begin{aligned} W_{14} &= W_{10} \text{ xor } W_{13} \\ 6C &= BE \text{ xor } D2 \\ 59 &= 9B \text{ xor } C2 \\ 0C &= C5 \text{ xor } C9 \\ BF &= 00 \text{ xor } BF \end{aligned} \qquad \begin{aligned} W_{15} &= W_{11} \text{ xor } W_{14} \\ 04 &= 68 \text{ xor } 6C \\ 69 &= 30 \text{ xor } 59 \\ BF &= B3 \text{ xor } 0C \\ 41 &= FE \text{ xor } BF \end{aligned}$$

Sub-key $K_3 = B6\,FF\,74\,4E\,D2\,C2\,C9\,BF\,6C\,59\,0C\,BF\,04\,69\,BF\,41$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{16} = W_{12} \text{ xor } \texttt{ByteSub}(S_1 W_{15})$$

$$\begin{aligned} 4F &= B6 \text{ xor } F9 \\ F7 &= FF \text{ xor } 08 \\ F7 &= 74 \text{ xor } 83 \\ BC &= 4E \text{ xor } F2 \end{aligned}$$

$$\begin{aligned} W_{16}[0] &= W_{16}[0] \text{ xor } rcon_4 \\ 47 &= 4F \text{ xor } 10 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{lcl|lcl|lcl} W_{17} & = & W_{13} \text{ xor } W_{16} & W_{18} & = & W_{14} \text{ xor } W_{17} & W_{19} & = & W_{15} \text{ xor } W_{18} \\ \\ 95 & = & D2 \text{ xor } 47 & F9 & = & 6C \text{ xor } 95 & FD & = & 04 \text{ xor } F9 \\ 35 & = & C2 \text{ xor } F7 & 6C & = & 59 \text{ xor } 35 & 05 & = & 69 \text{ xor } 6C \\ 3E & = & C9 \text{ xor } F7 & 32 & = & 0C \text{ xor } 3E & 8D & = & BF \text{ xor } 32 \\ 03 & = & BF \text{ xor } BC & BC & = & BF \text{ xor } 03 & FD & = & 41 \text{ xor } BC \end{array}$$

Sub-key $K_4 = 47\,F7\,F7\,BC\,95\,35\,3E\,03\,F9\,6C\,32\,BC\,FD\,05\,8D\,FD$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{20} = W_{16} \text{ xor } \texttt{ByteSub}(S_1 W_{19})$$

$$\begin{aligned} 2C &= 47 \text{ xor } 6B \\ AA &= F7 \text{ xor } 5D \\ A3 &= F7 \text{ xor } 54 \\ E8 &= BC \text{ xor } 54 \end{aligned}$$

$$\begin{aligned} W_{20}[0] &= W_{20}[0] \text{ xor } rcon_5 \\ 3C &= 2C \text{ xor } 20 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{lcl|lcl|lcl} W_{21} & = & W_{17} \text{ xor } W_{20} & W_{22} & = & W_{18} \text{ xor } W_{21} & W_{23} & = & W_{19} \text{ xor } W_{22} \\ \\ A9 & = & 95 \text{ xor } 3C & 50 & = & F9 \text{ xor } A9 & AD & = & FD \text{ xor } 50 \\ 9F & = & 35 \text{ xor } AA & F3 & = & 6C \text{ xor } 9F & F6 & = & 05 \text{ xor } F3 \\ 9D & = & 3E \text{ xor } A3 & AF & = & 32 \text{ xor } 9D & 22 & = & 8D \text{ xor } AF \\ EB & = & 03 \text{ xor } E8 & 57 & = & BC \text{ xor } EB & AA & = & FD \text{ xor } 57 \end{array}$$

Sub-key $K_5 = 3C\ AA\ A3\ E8\ A9\ 9F\ 9D\ EB\ 50\ F3\ AF\ 57\ AD\ F6\ 22\ AA$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{24} = W_{20} \text{ xor } \texttt{ByteSub}(S_1 W_{23})$$

$$\begin{aligned} 7E &= 3C \text{ xor } 42 \\ 39 &= AA \text{ xor } 93 \\ 0F &= A3 \text{ xor } AC \\ 7D &= E8 \text{ xor } 95 \end{aligned}$$

$$\begin{aligned} W_{24}[0] &= W_{24}[0] \text{ xor } rcon_6 \\ 5E &= 7E \text{ xor } 40 \end{aligned}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{aligned} W_{25} &= W_{21} \text{ xor } W_{24} & W_{26} &= W_{22} \text{ xor } W_{25} & W_{27} &= W_{23} \text{ xor } W_{26} \\ F7 &= A9 \text{ xor } 5E & A7 &= 50 \text{ xor } F7 & 0A &= AD \text{ xor } A7 \\ A6 &= 9F \text{ xor } 39 & 55 &= F3 \text{ xor } A6 & A3 &= F6 \text{ xor } 55 \\ 92 &= 9D \text{ xor } 0F & 3D &= AF \text{ xor } 92 & 1F &= 22 \text{ xor } 3D \\ 96 &= EB \text{ xor } 7D & C1 &= 57 \text{ xor } 96 & 6B &= AA \text{ xor } C1 \end{aligned}$$

Sub-key $K_6 = 5E\ 39\ 0F\ 7D\ F7\ A6\ 92\ 96\ A7\ 55\ 3D\ C1\ 0A\ A3\ 1F\ 6B$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{28} = W_{24} \text{ xor } \texttt{ByteSub}(S_1 W_{27})$$

$$\begin{aligned} 54 &= 5E \text{ xor } 0A \\ F9 &= 39 \text{ xor } C0 \\ 70 &= 0F \text{ xor } 7F \\ 1A &= 7D \text{ xor } 67 \end{aligned}$$

$$\begin{array}{rcl} W_{28}[0] & = & W_{28}[0] \text{ xor } rcon_7 \\ 14 & = & 54 \text{ xor } 80 \end{array}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{rcl@{\qquad}rcl@{\qquad}rcl} W_{29} & = & W_{25} \text{ xor } W_{28} & W_{30} & = & W_{26} \text{ xor } W_{29} & W_{31} & = & W_{27} \text{ xor } W_{30} \\ \\ E3 & = & F7 \text{ xor } 14 & 44 & = & A7 \text{ xor } E3 & 4E & = & 0A \text{ xor } 44 \\ 5F & = & A6 \text{ xor } F9 & 0A & = & 55 \text{ xor } 5F & A9 & = & A3 \text{ xor } 0A \\ E2 & = & 92 \text{ xor } 70 & DF & = & 3D \text{ xor } E2 & C0 & = & 1F \text{ xor } DF \\ 8C & = & 96 \text{ xor } 1A & 4D & = & C1 \text{ xor } 8C & 26 & = & 6B \text{ xor } 4D \end{array}$$

Sub-key $K_7 = 14\,F9\,70\,1A\,E3\,5F\,E2\,8C\,44\,0A\,DF\,4D\,4E\,A9\,C0\,26$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{32} = W_{28} \text{ xor } \texttt{ByteSub}(S_1 W_{31})$$

$$\begin{array}{rcl} C7 & = & 14 \text{ xor } D3 \\ 43 & = & F9 \text{ xor } BA \\ 87 & = & 70 \text{ xor } F7 \\ 35 & = & 1A \text{ xor } 2F \end{array}$$

$$\begin{array}{rcl} W_{32}[0] & = & W_{32}[0] \text{ xor } rcon_8 \\ 47 & = & C7 \text{ xor } 1B \end{array}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{rcl@{\qquad}rcl@{\qquad}rcl} W_{33} & = & W_{29} \text{ xor } W_{32} & W_{34} & = & W_{30} \text{ xor } W_{33} & W_{35} & = & W_{31} \text{ xor } W_{34} \\ \\ A4 & = & E3 \text{ xor } 47 & E0 & = & 44 \text{ xor } A4 & AE & = & 4E \text{ xor } E0 \\ 1C & = & 5F \text{ xor } 43 & 16 & = & 0A \text{ xor } 1C & BF & = & A9 \text{ xor } 16 \\ 65 & = & E2 \text{ xor } 87 & BA & = & DF \text{ xor } 65 & 7A & = & C0 \text{ xor } BA \\ B9 & = & 8C \text{ xor } 35 & F4 & = & 4D \text{ xor } B9 & D2 & = & 26 \text{ xor } F4 \end{array}$$

Sub-key $K_8 = 47\,43\,87\,35\,A4\,1C\,65\,B9\,E0\,16\,BA\,F4\,AE\,BF\,7A\,D2$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{36} = W_{32} \text{ xor } \texttt{ByteSub}(S_1 W_{34})$$

$$\begin{array}{rcl} 4F & = & 47 \text{ xor } 08 \\ 99 & = & 43 \text{ xor } DA \\ 32 & = & 87 \text{ xor } B5 \\ D1 & = & 35 \text{ xor } E4 \end{array}$$

$$\begin{array}{rcl} W_{36}[0] & = & W_{36}[0] \text{ xor } rcon_9 \\ 54 & = & 4F \text{ xor } 36 \end{array}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{rcl} W_{37} & = & W_{33} \text{ xor } W_{36} \\ F0 & = & A4 \text{ xor } 54 \\ 85 & = & 1C \text{ xor } 99 \\ 57 & = & 65 \text{ xor } 32 \\ 68 & = & B9 \text{ xor } D1 \end{array} \qquad \begin{array}{rcl} W_{38} & = & W_{34} \text{ xor } W_{37} \\ 10 & = & E0 \text{ xor } F0 \\ 93 & = & 16 \text{ xor } 85 \\ ED & = & BA \text{ xor } 57 \\ 9C & = & F4 \text{ xor } 68 \end{array} \qquad \begin{array}{rcl} W_{39} & = & W_{35} \text{ xor } W_{38} \\ BE & = & AE \text{ xor } 10 \\ 2C & = & BF \text{ xor } 93 \\ 97 & = & 7A \text{ xor } ED \\ 4E & = & D2 \text{ xor } 9C \end{array}$$

Sub-key $K_9 = 54\,99\,32\,D1\,F0\,85\,57\,68\,10\,93\,ED\,9C\,BE\,2C\,97\,4E$

Evaluation of W_i when i is multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } \texttt{ByteSub}(S_1 W_{i-1}) \text{ xor } rcon_k$$

and the calculated values are:

$$W_{40} = W_{36} \text{ xor } \texttt{ByteSub}(S_1 W_{39})$$

$$\begin{array}{rcl} 25 & = & 54 \text{ xor } 71 \\ 11 & = & 99 \text{ xor } 88 \\ 1D & = & 32 \text{ xor } 2F \\ 7F & = & D1 \text{ xor } AE \end{array}$$

$$\begin{array}{rcl} W_{40}[0] & = & W_{40}[0] \text{ xor } rcon_{10} \\ 13 & = & 25 \text{ xor } 6C \end{array}$$

Evaluation of W_i when i is not a multiple of 4. It is obtained using the rule

$$W_i = W_{i-4} \text{ xor } W_{i-1}$$

$$\begin{array}{rcl} W_{41} & = & W_{37} \text{ xor } W_{40} \\ E3 & = & F0 \text{ xor } 13 \\ 94 & = & 85 \text{ xor } 11 \\ 4A & = & 57 \text{ xor } 1D \\ 17 & = & 68 \text{ xor } 7F \end{array} \qquad \begin{array}{rcl} W_{42} & = & W_{38} \text{ xor } W_{41} \\ F3 & = & 10 \text{ xor } E3 \\ 07 & = & 93 \text{ xor } 94 \\ A7 & = & ED \text{ xor } 4A \\ 8B & = & 9C \text{ xor } 17 \end{array} \qquad \begin{array}{rcl} W_{43} & = & W_{39} \text{ xor } W_{42} \\ 4D & = & BE \text{ xor } F3 \\ 2B & = & 2C \text{ xor } 07 \\ 30 & = & 97 \text{ xor } A7 \\ C5 & = & 4E \text{ xor } 8B \end{array}$$

Sub-key $K_1 0 = 13\,11\,1D\,7F\,E3\,94\,4A\,17\,F3\,07\,A7\,8B\,4D\,2B\,30\,C5$

3 Encription

Initial Round. Key Addition.

$$\begin{array}{ccc} B_{ji} & & K_0[j,i] \\ \begin{pmatrix} 00 & 04 & 08 & 0C \\ 01 & 05 & 09 & 0D \\ 02 & 06 & 0A & 0E \\ 03 & 07 & 0B & 0F \end{pmatrix} & \text{xor} & \begin{pmatrix} 00 & 04 & 08 & 0C \\ 01 & 05 & 09 & 0D \\ 02 & 06 & 0A & 0E \\ 03 & 07 & 0B & 0F \end{pmatrix} \end{array} = \begin{array}{c} B_{ji} \\ \begin{pmatrix} 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \end{pmatrix} \end{array}$$

This transformation applies a xor in every entry of the matrix B_{ji} and $K_0[ji]$, i.e. $B_{ji} = B_{ij}$ xor $K_0[j,i]$.

Round 1. Byte Substitution.

$$\begin{pmatrix} 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \\ 00 & 00 & 00 & 00 \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the

irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Linear Transformation LT

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j,i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 1. ShiftRow.

$$\left(\begin{array}{cccc} 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \end{array}\right) \qquad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \\ \hline 63 & 63 & 63 & 63 \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 1. Mix-Column

$$\left(\begin{array}{c|c|c|c} 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \end{array}\right) \qquad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 1. Key Addition.

$$\begin{pmatrix} 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \\ 63 & 63 & 63 & 63 \end{pmatrix} \text{ xor } \begin{pmatrix} D6 & D2 & DA & D6 \\ AA & AF & A6 & AB \\ 74 & 72 & 78 & 76 \\ FD & FA & F1 & FE \end{pmatrix} = \begin{pmatrix} B5 & B1 & B9 & B5 \\ C9 & CC & C5 & C8 \\ 17 & 11 & 1B & 15 \\ 9E & 99 & 92 & 9D \end{pmatrix}$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_1 = B5\, C9\, 17\, 9E\, B1\, CC\, 11\, 99\, B9\, C5\, 1B\, 92\, B5\, C8\, 15\, 9D$$

Round 2. Byte Substitution.

$$\begin{pmatrix} B5 & B1 & B9 & B5 \\ C9 & CC & C5 & C8 \\ 17 & 11 & 1B & 15 \\ 9E & 99 & 92 & 9D \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} 75 & E0 & 8E & 75 \\ 27 & 1B & D4 & A9 \\ 5F & B4 & CC & 2B \\ 89 & 14 & 32 & DC \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 2. Linear Transformation LT.

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} D5 & C8 & 56 & D5 \\ DD & 4B & A6 & E8 \\ F0 & 82 & AF & 59 \\ 0B & EE & 4F & 5E \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j, i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 2. ShiftRow.

$$\begin{pmatrix} D5 & C8 & 56 & D5 \\ \hline DD & 4B & A6 & E8 \\ \hline F0 & 82 & AF & 59 \\ \hline 0B & EE & 4F & 5E \end{pmatrix} \quad \texttt{ShiftRow}(a_j) = \begin{pmatrix} D5 & C8 & 56 & D5 \\ \hline 4B & A6 & E8 & DD \\ \hline AF & 59 & F0 & 82 \\ \hline 5E & 0B & EE & 4F \end{pmatrix}$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 2. Mix-Column

$$\left(\begin{array}{c|c|c|c} D5 & C8 & 56 & D5 \\ 4B & A6 & E8 & DD \\ AF & 59 & F0 & 82 \\ 5E & 0B & EE & 4F \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 9D & 28 & 91 & 00 \\ F7 & 7F & 78 & A6 \\ 39 & C1 & 6C & C6 \\ 3C & AA & 25 & A5 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 2. Key Addition.

$$\begin{pmatrix} 9D & 28 & 91 & 00 \\ F7 & 7F & 78 & A6 \\ 39 & C1 & 6C & C6 \\ 3C & AA & 25 & A5 \end{pmatrix} \text{ xor } \begin{pmatrix} B6 & 64 & BE & 68 \\ 92 & 3D & 9B & 30 \\ CF & BD & C5 & B3 \\ 0B & F1 & 00 & FE \end{pmatrix} = \begin{pmatrix} 2B & 4C & 2F & 68 \\ 65 & 42 & E3 & 96 \\ F6 & 7C & A9 & 75 \\ 37 & 5B & 25 & 5B \end{pmatrix}$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j, i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j, i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_2 = 2B\,65\,F6\,37\,4C\,42\,7C\,5B\,2F\,E3\,A9\,25\,68\,96\,75\,5B$$

Round 3. Byte Substitution.

$$\begin{pmatrix} 2B & 4C & 2F & 68 \\ 65 & 42 & E3 & 96 \\ F6 & 7C & A9 & 75 \\ 37 & 5B & 25 & 5B \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} 15 & 54 & C2 & F4 \\ A6 & 37 & EB & 84 \\ 03 & A1 & C8 & B5 \\ 42 & F0 & 4D & F0 \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 3. Linear Transformation LT.

Here is the calculation for the entry $(3, 3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} F1 & 29 & 15 & 45 \\ 4D & 2C & 11 & 90 \\ 42 & 10 & D3 & 9D \\ 9A & 39 & 3F & 39 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j, i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 3. ShiftRow.

$$\left(\begin{array}{cccc} F1 & 29 & 15 & 45 \\ \hline 4D & 2C & 11 & 90 \\ \hline 42 & 10 & D3 & 9D \\ \hline 9A & 39 & 3F & 39 \end{array}\right) \qquad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} F1 & 29 & 15 & 45 \\ \hline 2C & 11 & 90 & 4D \\ \hline D3 & 9D & 42 & 10 \\ \hline 39 & 9A & 39 & 3F \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of

192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 3. Mix-Column

$$\left(\begin{array}{c|c|c|c} F1 & 29 & 15 & 45 \\ 2C & 11 & 90 & 4D \\ D3 & 9D & 42 & 10 \\ 39 & 9A & 39 & 3F \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 67 & 66 & FA & 72 \\ FE & 2D & D1 & D0 \\ 2B & AC & 4A & 69 \\ 85 & D8 & 9F & EC \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 3. Key Addition.

$$\left(\begin{array}{cccc} 67 & 66 & FA & 72 \\ FE & 2D & D1 & D0 \\ 2B & AC & 4A & 69 \\ 85 & D8 & 9F & EC \end{array}\right) \text{ xor } \left(\begin{array}{cccc} B6 & D2 & 6C & 04 \\ FF & C2 & 59 & 69 \\ 74 & C9 & 0C & BF \\ 4E & BF & BF & 41 \end{array}\right) = \left(\begin{array}{cccc} D1 & B4 & 96 & 76 \\ 01 & EF & 88 & B9 \\ 5F & 65 & 46 & D6 \\ CB & 67 & 20 & AD \end{array}\right)$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j, i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j, i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_3 = D1\,01\,5F\,CB\,B4\,EF\,65\,67\,96\,88\,46\,20\,76\,B9\,D6\,AD$$

Round 4. Byte Substitution.

$$\left(\begin{array}{cccc} D1 & B4 & 96 & 76 \\ 01 & EF & 88 & B9 \\ 5F & 65 & 46 & D6 \\ CB & 67 & 20 & AD \end{array}\right) \quad a_{ij}^{-1} \quad = \quad \left(\begin{array}{cccc} 07 & 11 & 84 & BA \\ 01 & B3 & 9B & 8E \\ 17 & A6 & F5 & E2 \\ 04 & 43 & 3A & E7 \end{array}\right)$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 4. Linear Transformation LT.

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} 3E & 8D & 90 & 38 \\ 7C & DF & C4 & 56 \\ CF & 4D & 5A & F6 \\ 1F & 85 & B7 & 95 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j,i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 4. ShiftRow.

$$\begin{pmatrix} 3E & 8D & 90 & 38 \\ 7C & DF & C4 & 56 \\ CF & 4D & 5A & F6 \\ 1F & 85 & B7 & 95 \end{pmatrix} \qquad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 3E & 8D & 90 & 38 \\ \hline DF & C4 & 56 & 7C \\ \hline 5A & F6 & CF & 4D \\ \hline 95 & 1F & 85 & B7 \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 4. Mix-Column

$$\left(\begin{array}{c|c|c|c} 3E & 8D & 90 & 38 \\ DF & C4 & 56 & 7C \\ 5A & F6 & CF & 4D \\ 95 & 1F & 85 & B7 \end{array}\right) \qquad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} C9 & BF & 8B & 0E \\ E0 & 00 & F3 & A0 \\ F1 & 9F & D7 & 1C \\ F6 & 80 & 23 & 0C \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 4. Key Addition.

$$\begin{pmatrix} C9 & BF & 8B & 0E \\ E0 & 00 & F3 & A0 \\ F1 & 9F & D7 & 1C \\ F6 & 80 & 23 & 0C \end{pmatrix} \text{ xor } \begin{pmatrix} 47 & 95 & F9 & FD \\ F7 & 35 & 6C & 05 \\ F7 & 3E & 32 & 8D \\ BC & 03 & BC & FD \end{pmatrix} = \begin{pmatrix} 8E & 2A & 72 & F3 \\ 17 & 35 & 9F & A5 \\ 06 & A1 & E5 & 91 \\ 4A & 83 & 9F & F1 \end{pmatrix}$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_4 = 8E\,17\,06\,4A\,2A\,35\,A1\,83\,72\,9F\,E5\,9F\,F3\,A5\,91\,F1$$

Round 5. Byte Substitution.

$$\begin{pmatrix} 8E & 2A & 72 & F3 \\ 17 & 35 & 9F & A5 \\ 06 & A1 & E5 & 91 \\ 4A & 83 & 9F & F1 \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} B9 & 98 & 97 & 34 \\ 5F & 39 & 9A & B8 \\ 7B & 7C & 0E & 6A \\ AB & 80 & 9A & 23 \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 5. Linear Transformation LT.

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} 19 & E5 & 40 & 0D \\ F0 & 96 & DB & 06 \\ 6F & 32 & D9 & 81 \\ D6 & EC & DB & A1 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j,i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 5. ShiftRow.

$$\left(\begin{array}{cccc} 19 & E5 & 40 & 0D \\ \hline F0 & 96 & DB & 06 \\ \hline 6F & 32 & D9 & 81 \\ \hline D6 & EC & DB & A1 \end{array}\right) \quad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 19 & E5 & 40 & 0D \\ \hline 96 & DB & 06 & F0 \\ \hline D9 & 81 & 6F & 32 \\ \hline A1 & D6 & EC & DB \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 5. Mix-Column

$$\left(\begin{array}{c|c|c|c} 19 & E5 & 40 & 0D \\ 96 & DB & 06 & F0 \\ D9 & 81 & 6F & 32 \\ A1 & D6 & EC & DB \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} EB & F0 & 09 & F8 \\ FF & 06 & 11 & 7B \\ DE & 46 & B7 & EF \\ 3D & D9 & 6A & 78 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 5. Key Addition.

$$\left(\begin{array}{cccc} EB & F0 & 09 & F8 \\ FF & 06 & 11 & 7B \\ DE & 46 & B7 & EF \\ 3D & D9 & 6A & 78 \end{array}\right) \text{ xor } \left(\begin{array}{cccc} 3C & A9 & 50 & AD \\ AA & 9F & F3 & F6 \\ A3 & 9D & AF & 22 \\ E8 & EB & 57 & AA \end{array}\right) = \left(\begin{array}{cccc} D7 & 59 & 59 & 55 \\ 55 & 99 & E2 & 8D \\ 7D & DB & 18 & CD \\ D5 & 32 & 3D & D2 \end{array}\right)$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_5 = D7\,55\,7D\,D5\,59\,99\,DB\,32\,59\,E2\,18\,3D\,55\,8D\,CD\,D2$$

Round 6. Byte Substitution.

$$\begin{pmatrix} D7 & 59 & 59 & 55 \\ 55 & 99 & E2 & 8D \\ 7D & DB & 18 & CD \\ D5 & 32 & 3D & D2 \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} EA & 3E & 3E & 24 \\ 24 & 14 & D6 & 02 \\ FA & D5 & 58 & FC \\ DB & 92 & BB & AE \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 6. Linear Transformation LT.

Here is the calculation for the entry $(3, 3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} 0E & CB & CB & FC \\ FC & EE & 98 & 5D \\ FF & B9 & AD & BD \\ 03 & 23 & 27 & B5 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j, i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 6. ShiftRow.

$$\left(\begin{array}{cccc} 0E & CB & CB & FC \\ \hline FC & EE & 98 & 5D \\ \hline FF & B9 & AD & BD \\ \hline 03 & 23 & 27 & B5 \end{array}\right) \quad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 0E & CB & CB & FC \\ \hline EE & 98 & 5D & FC \\ \hline AD & BD & FF & B9 \\ \hline B5 & 03 & 23 & 27 \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of

192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 6. Mix-Column

$$\left(\begin{array}{c|c|c|c} 0E & CB & CB & FC \\ EE & 98 & 5D & FC \\ AD & BD & FF & B9 \\ B5 & 03 & 23 & 27 \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 2D & 80 & B6 & 62 \\ 90 & 3F & 48 & E8 \\ 65 & 37 & 16 & 00 \\ 20 & 65 & A2 & 14 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 6. Key Addition.

$$\left(\begin{array}{cccc} 2D & 80 & B6 & 62 \\ 90 & 3F & 48 & E8 \\ 65 & 37 & 16 & 00 \\ 20 & 65 & A2 & 14 \end{array}\right) \text{ xor } \left(\begin{array}{cccc} 5E & F7 & A7 & 0A \\ 39 & A6 & 55 & A3 \\ 0F & 92 & 3D & 1F \\ 7D & 96 & C1 & 6B \end{array}\right) = \left(\begin{array}{cccc} 73 & 77 & 11 & 68 \\ A9 & 99 & 1D & 4B \\ 6A & A5 & 2B & 1F \\ 5D & F3 & 63 & 7F \end{array}\right)$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j, i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j, i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_6 = 73\ A9\ 6A\ 5D\ 77\ 99\ A5\ F3\ 11\ 1D\ 2B\ 63\ 68\ 4B\ 1F\ 7F$$

Round 7. Byte Substitution.

$$\left(\begin{array}{cccc} 73 & 77 & 11 & 68 \\ A9 & 99 & 1D & 4B \\ 6A & A5 & 2B & 1F \\ 5D & F3 & 63 & 7F \end{array}\right) \quad a_{ij}^{-1} \quad = \quad \left(\begin{array}{cccc} 85 & 3C & B4 & F4 \\ C8 & 14 & 40 & 13 \\ 91 & B8 & 15 & B2 \\ EC & 34 & D3 & 82 \end{array}\right)$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 7. Linear Transformation LT.

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} 8F & F5 & 82 & 45 \\ D3 & EE & A4 & B3 \\ 02 & 06 & F1 & C0 \\ 4C & 0D & FB & D2 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j,i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 7. ShiftRow.

$$\left(\begin{array}{cccc} 8F & F5 & 82 & 45 \\ \hline D3 & EE & A4 & B3 \\ \hline 02 & 06 & F1 & C0 \\ \hline 4C & 0D & FB & D2 \end{array}\right) \qquad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 8F & F5 & 82 & 45 \\ \hline EE & A4 & B3 & D3 \\ \hline F1 & C0 & 02 & 06 \\ \hline D2 & 4C & 0D & FB \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 7. Mix-Column

$$\left(\begin{array}{c|c|c|c} 8F & F5 & 82 & 45 \\ EE & A4 & B3 & D3 \\ F1 & C0 & 02 & 06 \\ D2 & 4C & 0D & FB \end{array}\right) \qquad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 0F & 8A & DE & 19 \\ 92 & B1 & F4 & 09 \\ F5 & 1E & 22 & 8C \\ 2A & F8 & 36 & F7 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 7. Key Addition.

$$\begin{pmatrix} 0F & 8A & DE & 19 \\ 92 & B1 & F4 & 09 \\ F5 & 1E & 22 & 8C \\ 2A & F8 & 36 & F7 \end{pmatrix} \text{ xor } \begin{pmatrix} 14 & E3 & 44 & 4E \\ F9 & 5F & 0A & A9 \\ 70 & E2 & DF & C0 \\ 1A & 8C & 4D & 26 \end{pmatrix} = \begin{pmatrix} 1B & 69 & 9A & 57 \\ 6B & EE & FE & A0 \\ 85 & FC & FD & 4C \\ 30 & 74 & 7B & D1 \end{pmatrix}$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_7 = 1B\,6B\,85\,30\,69\,EE\,FC\,74\,9A\,FE\,FD\,7B\,57\,A0\,4C\,D1$$

Round 8. Byte Substitution.

$$\begin{pmatrix} 1B & 69 & 9A & 57 \\ 6B & EE & FE & A0 \\ 85 & FC & FD & 4C \\ 30 & 74 & 7B & D1 \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} CC & 47 & 9F & BF \\ DF & 1E & 41 & FB \\ 73 & CD & 1A & 54 \\ 2C & 10 & 06 & 07 \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 8. Linear Transformation LT.

Here is the calculation for the entry $(3,3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} AF & F9 & B8 & 5B \\ 7F & 28 & BB & E0 \\ 97 & B0 & 54 & 29 \\ 04 & 92 & 21 & 3E \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j,i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 8. ShiftRow.

$$\left(\begin{array}{cccc} AF & F9 & B8 & 5B \\ \hline 7F & 28 & BB & E0 \\ \hline 97 & B0 & 54 & 29 \\ \hline 04 & 92 & 21 & 3E \end{array}\right) \quad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} AF & F9 & B8 & 5B \\ \hline 28 & BB & E0 & 7F \\ \hline 54 & 29 & 97 & B0 \\ \hline 3E & 04 & 92 & 21 \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of 192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 8. Mix-Column

$$\left(\begin{array}{c|c|c|c} AF & F9 & B8 & 5B \\ 28 & BB & E0 & 7F \\ 54 & 29 & 97 & B0 \\ 3E & 04 & 92 & 21 \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} 57 & 12 & 55 & A6 \\ 3D & EB & 53 & 4F \\ 6D & 1C & C0 & 3C \\ EA & 8A & 9B & 60 \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 8. Key Addition.

$$\left(\begin{array}{cccc} 57 & 12 & 55 & A6 \\ 3D & EB & 53 & 4F \\ 6D & 1C & C0 & 3C \\ EA & 8A & 9B & 60 \end{array}\right) \text{ xor } \left(\begin{array}{cccc} 47 & A4 & E0 & AE \\ 43 & 1C & 16 & BF \\ 87 & 65 & BA & 7A \\ 35 & B9 & F4 & D2 \end{array}\right) = \left(\begin{array}{cccc} 10 & B6 & B5 & 08 \\ 7E & F7 & 45 & F0 \\ EA & 79 & 7A & 46 \\ DF & 33 & 6F & B2 \end{array}\right)$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_8 = 10\,7E\,EA\,DF\,B6\,F7\,79\,33\,B5\,45\,7A\,6F\,08\,F0\,46\,B2$$

Round 9. Byte Substitution.

$$\begin{pmatrix} 10 & B6 & B5 & 08 \\ 7E & F7 & 45 & F0 \\ EA & 79 & 7A & 46 \\ DF & 33 & 6F & B2 \end{pmatrix} \quad a_{ij}^{-1} \quad = \quad \begin{pmatrix} 74 & 78 & 75 & E8 \\ 81 & 8C & 31 & 5B \\ D7 & 70 & D0 & F5 \\ 6B & 6C & 3B & 1F \end{pmatrix}$$

This transformation replace every entry of the input B_{ji} by the result of two operations: The first one maps an entry to their multiplicative inverse in $GF(2^8)$. Every entry (a byte) have a polynomial representation with the irreducible polynomial $f(x) = x^8 + x^4 + x^3 + x + 1$. The zero is mapped to zero.

Round 8. Linear Transformation LT.

Here is the calculation for the entry $(3, 3)$, $LT(a_{33}^{-1})$:

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{pmatrix} = \begin{pmatrix} 10001111 \\ 11000111 \\ 11100011 \\ 11110001 \\ 11111000 \\ 01111100 \\ 00111110 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad LT(a_{ji}^{-1}) = \begin{pmatrix} CA & 4E & D5 & 30 \\ F3 & 68 & 6E & 8C \\ 87 & B6 & DA & 5A \\ 9E & C3 & A8 & 37 \end{pmatrix}$$

The second operation apply a linear transformation from $GF(2^8)$ to $GF(2^8)$ in every entry (byte) of the matrix. The input are the bits $x_7, \ldots, x_0$ in a_{ji}^{-1} and the output are $y_7, \ldots, y0$, this is the new (j, i) element B_{ji}.

This is the final output in the transformation `ByteSub` .

Round 9. ShiftRow.

$$\left(\begin{array}{cccc} CA & 4E & D5 & 30 \\ \hline F3 & 68 & 6E & 8C \\ \hline 87 & B6 & DA & 5A \\ \hline 9E & C3 & A8 & 37 \end{array}\right) \qquad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} CA & 4E & D5 & 30 \\ \hline 68 & 6E & 8C & F3 \\ \hline DA & 5A & 87 & B6 \\ \hline 37 & 9E & C3 & A8 \end{array}\right)$$

This transformation is applied to the rows of B_{ji}, it shifts bytes of the rows.

The row 1 is not shifted, the row 2 is shifted 1 byte, the row 3 shifted 2 bytes and the row 4, 3 bytes (as is in our case of 128 bits). In the case of

192 bits is the same, and the case of 256 bits only changes the row 3,4, that are shifted 3 and 4 bytes respectively.

Round 9. Mix-Column

$$\left(\begin{array}{c|c|c|c} CA & 4E & D5 & 30 \\ 68 & 6E & 8C & F3 \\ DA & 5A & 87 & B6 \\ 37 & 9E & C3 & A8 \end{array}\right) \quad \texttt{MixColumn}(a_j) = \left(\begin{array}{c|c|c|c} DA & EA & 7A & 70 \\ 58 & E2 & 87 & A4 \\ 54 & 2D & 12 & 57 \\ 99 & C1 & F2 & 5E \end{array}\right)$$

This transformation is applied to the columns of B_{ji}. We can see each column as a polynomial of $GF(2^8)[x]$, so `MixColumn` multiply each column by the constant polynomial $c(x) = 03x^3 + 01x^2 + 01x + 02$, modulus $x^4 + 1$ their coefficients are in hexadecimal, and they are in $GF(2^8)$.

Round 9. Key Addition.

$$\left(\begin{array}{cccc} DA & EA & 7A & 70 \\ 58 & E2 & 87 & A4 \\ 54 & 2D & 12 & 57 \\ 99 & C1 & F2 & 5E \end{array}\right) \text{ xor } \left(\begin{array}{cccc} 54 & F0 & 10 & BE \\ 99 & 85 & 93 & 2C \\ 32 & 57 & ED & 97 \\ D1 & 68 & 9C & 4E \end{array}\right) = \left(\begin{array}{cccc} 8E & 1A & 6A & CE \\ C1 & 67 & 14 & 88 \\ 66 & 7A & FF & C0 \\ 48 & A9 & 6E & 10 \end{array}\right)$$

This transformation only Xor-ed every entry the matrix B_{ji} by every entry of $K_r[j,i]$ from the Key schedule. $B_{ji} = B_{ji}$ xor $K_r[j,i]$. This is the final of the round r, and the parcial vector test CT_r is

$$CT_9 = 8E\,C1\,66\,48\,1A\,67\,7A\,A9\,6A\,14\,FF\,6E\,CE\,88\,C0\,10$$

Final Round. Byte Substitution.

$$\left(\begin{array}{cccc} 8E & 1A & 6A & CE \\ C1 & 67 & 14 & 88 \\ 66 & 7A & FF & C0 \\ 48 & A9 & 6E & 10 \end{array}\right) \quad a_{ij}^{-1} \quad = \quad \left(\begin{array}{cccc} 19 & A2 & 02 & 8B \\ 78 & 85 & FA & C4 \\ 33 & DA & 16 & BA \\ 52 & D3 & 9F & CA \end{array}\right)$$

Final Round. ShiftRow.

$$\left(\begin{array}{cccc} 19 & A2 & 02 & 8B \\ \hline 78 & 85 & FA & C4 \\ \hline 33 & DA & 16 & BA \\ \hline 52 & D3 & 9F & CA \end{array}\right) \quad \texttt{ShiftRow}(a_j) = \left(\begin{array}{cccc} 19 & A2 & 02 & 8B \\ \hline 85 & FA & C4 & 78 \\ \hline 16 & BA & 33 & DA \\ \hline CA & 52 & D3 & 9F \end{array}\right)$$

Final Round. Key Addition.

$$\begin{pmatrix} 19 & A2 & 02 & 8B \\ 85 & FA & C4 & 78 \\ 16 & BA & 33 & DA \\ CA & 52 & D3 & 9F \end{pmatrix} \text{ xor } \begin{pmatrix} 13 & E3 & F3 & 4D \\ 11 & 94 & 07 & 2B \\ 1D & 4A & A7 & 30 \\ 7F & 17 & 8B & C5 \end{pmatrix} = \begin{pmatrix} 0A & 41 & F1 & C6 \\ 94 & 6E & C3 & 53 \\ 0B & F0 & 94 & EA \\ B5 & 45 & 58 & 5A \end{pmatrix}$$

$$Cipher\,Text = 0A\ 94\ 0B\ B5\ 41\ 6E\ F0\ 45\ F1\ C3\ 94\ 58\ C6\ 53\ EA\ 5A$$

In the decipher process, we apply the inverse transformations.

$$Decipher\,Text = 00\ 01\ 02\ 03\ 04\ 05\ 06\ 07\ 08\ 09\ 0A\ 0B\ 0C\ 0D\ 0E\ 0F$$